14.1 Intro Surfaces and Level Curves

Def'n:

A function, *f*, of two variables assigns a number for each input (x,y)

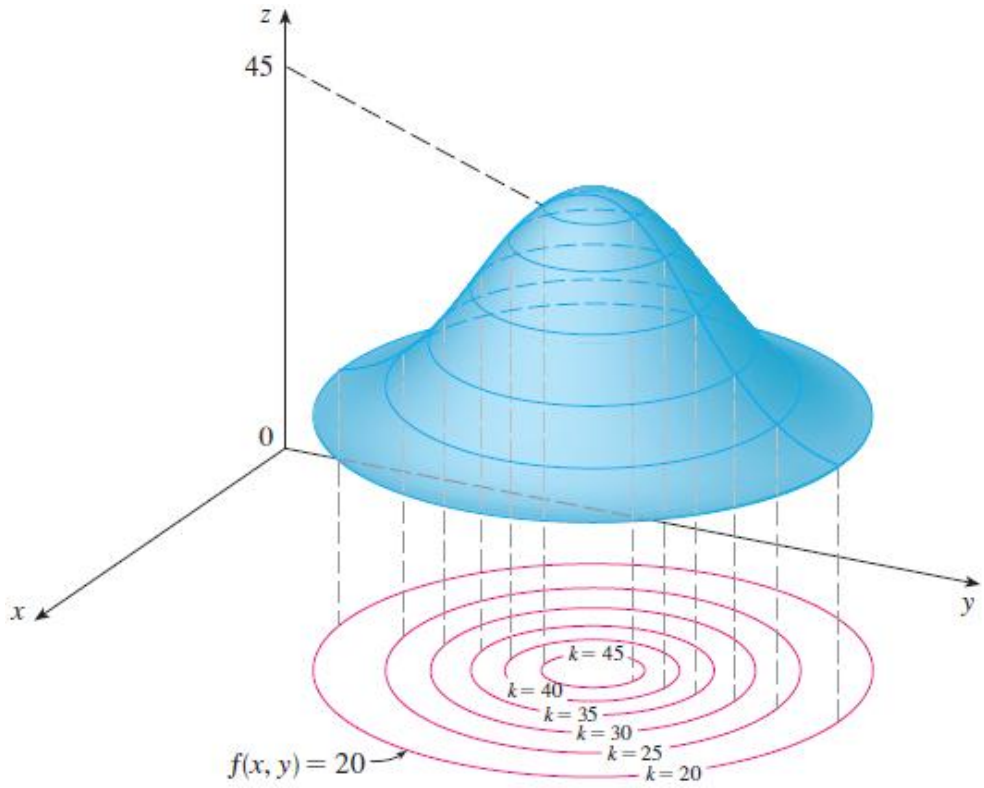
$$z = f(x, y).$$

(x,y) = location on the xy-plane.

z = f(x,y) = height above that pt.

We sometimes write $f: \mathbb{R}^2 \to \mathbb{R}$.

Our main visualization tool is *level curves* (traces for *z = k,* a constant)



Domain: *HW Q1-3*

The set of allowable inputs.

This will be a *region* in 2D.

Appears in Function	***Restriction***
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH ≠ 0
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	-1 ≤ BLAH ≤ 1
and other trig…	

Examples:

Sketch the domain of

(1) $f(x,y) = \ln(y - x)$

(2) $g(x,y) = \sqrt{y + x^2}$

Domain Graphs: https://www.desmos.com/3d/bqztlhjibs

Level Curves (*also called contour maps****)***

- Given $z = f(x, y)$
- Choose equally spaced-out values for z = k.
  - For each k value, graph the 2D curves $f(x, y) = k$ on the ground.

Examples: *HW Q4-6*

(1) Graph level curves for

$$z = \sin(x) - y$$

(2) Graph level curves for

$$z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

Drawing Level Curves: *https://www.desmos.com/3d/2iu7hz8kim*

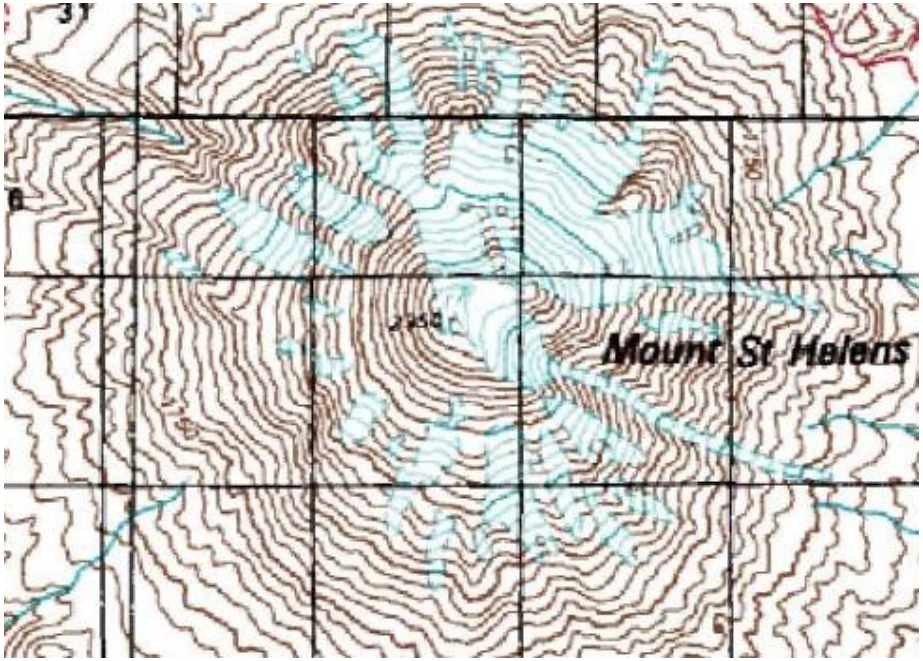
You have likely seen contour maps before

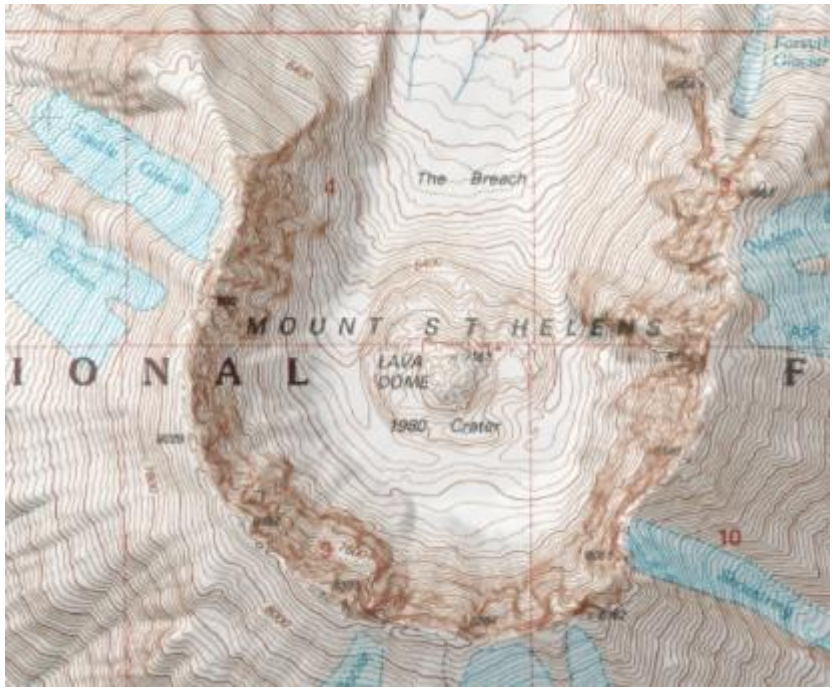
Here is a fun one…

Mount Saint Helen's before and after 1980



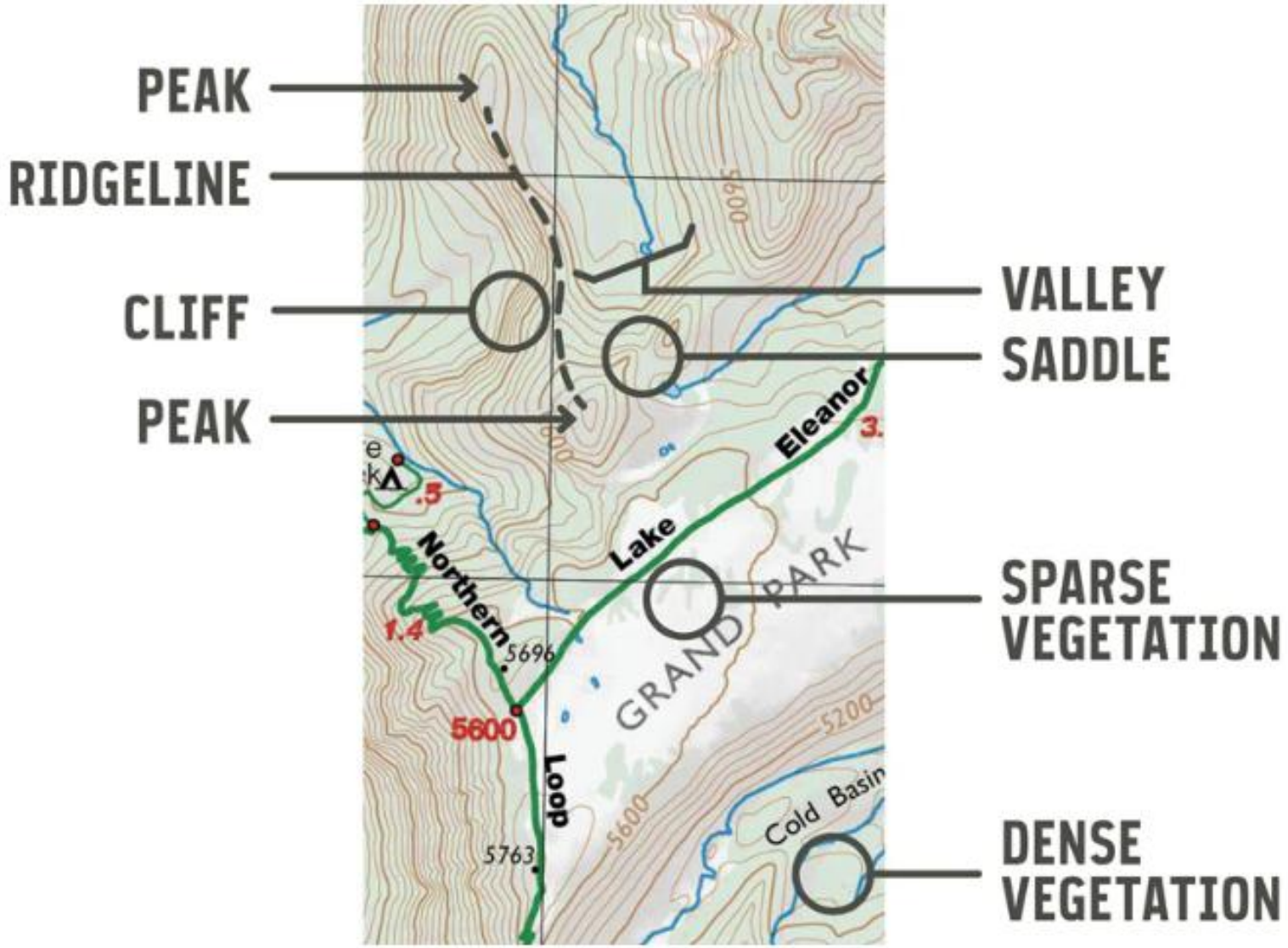






If you are having trouble with reading contour maps, then I encourage you to go to the REI website and they can teach you… *HW Q8-13*

https://www.rei.com/learn/expert-advice/topo-maps-how-to-use.html



Tips for identifying graphs in HW:

1. Plot points: f(0,0), f(0,1), f(1,0)
2. Draw level curve: z = 0, z = 1, z = 2
3. Special features:
   - Places undefined?
   - Behavior as x and y get bigger
   - Is it a wave if x (or y) is fixed?

Example: *HW Q7*

Match the function to the picture

A) $f(x,y) = \frac{1}{1+x^2+y^2}$

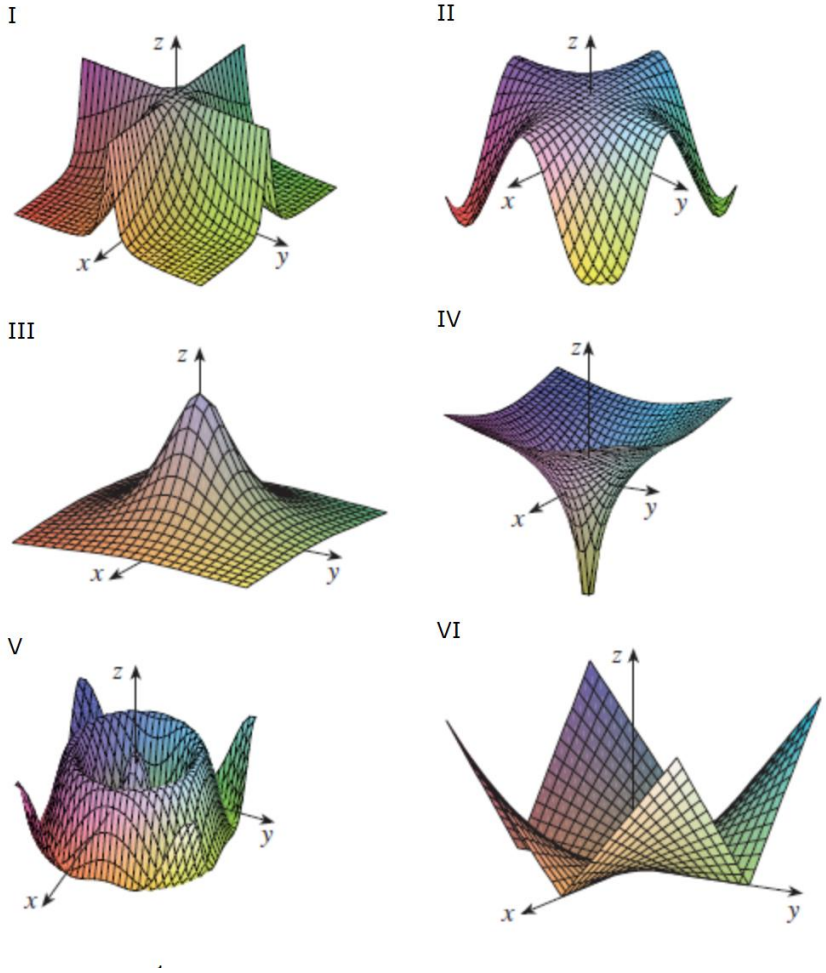
B) $f(x,y) = \frac{1}{1+x^2y^2}$

C) $f(x,y) = \ln(x^2+y^2)$

D) $f(x,y) = \cos\left(\sqrt{x^2+y^2}\right)$

E) $f(x,y) = |xy|$

F) $f(x,y) = \cos(xy)$



Rippling Wave: https://www.desmos.com/3d/2hb5veannv

A motivating contour map

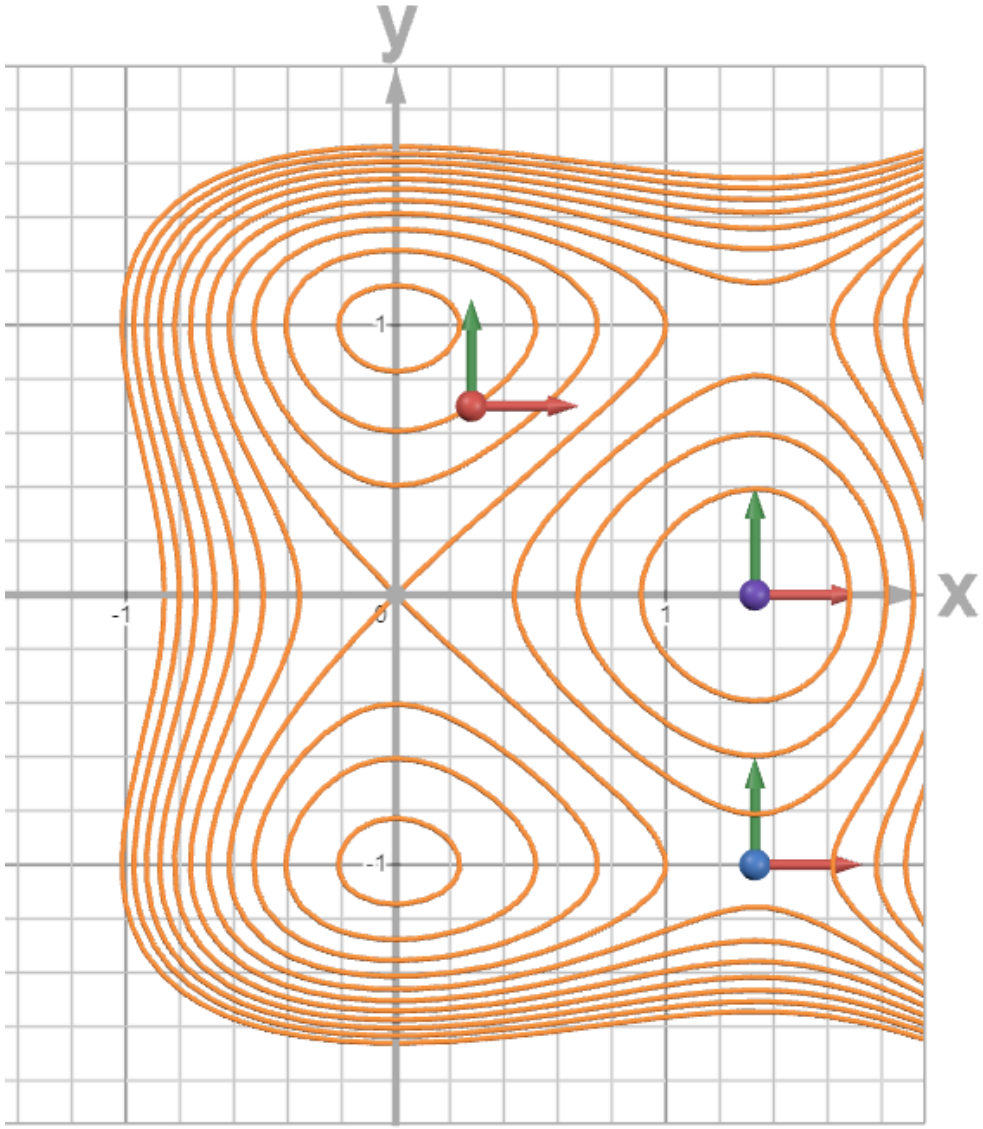
Visual https://www.desmos.com/3d/432nc4iwoo

Questions:

If we walk along the surface from a given point in one of the indicated directions, will we walk uphill or downhill?

What would you need to know to answer?

Does it matter which way we go?



14.3 Partial Derivatives

Intro to Partial Derivatives: https://www.desmos.com/3d/fwmtqk43et

Goal: Get slope in two different directions.

$$\frac{\partial z}{\partial x} = f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$\frac{\partial z}{\partial y} = f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

Example

$$f(x,y) = x^2y - 9y - xy^2 + y^3$$